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Temporal $1/f^{\alpha}$ Fluctuations from Fractal Magnetic Fields in Black Hole Accretion Flow

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Abstract

Rapid fluctuation with a frequency dependence of $1/f^{\alpha}$ (with $\alpha \simeq 1-2$) is characteristic of radiation from black-hole objects. Its origin remains poorly understood. We examine the three-dimensional magnetohydrodynamical (MHD) simulation data, finding that a magnetized accretion disk exhibits both $1/f^{\alpha}$ fluctuation (with $\alpha \simeq 2$) and a fractal magnetic structure (with the fractal dimension of $D \sim 1.9$). The fractal field configuration leads reconnection events with a variety of released energy and of duration, thereby producing $1/f^{\alpha}$ fluctuations.

Key words: Accretion, accretion disks — Advection-dominated flow — Black holes — Fractal — Magnetohydrodynamics

1. Introduction

Apparently random temporal fluctuations from Galactic black-hole candidates (BHCs; van der Klis 1995) and from active galactic nuclei (AGNs; Ulrich, Maraschi & Urry 1997) have led many astronomers to recognizing how complex the nature behaves. The light curves are neither periodic nor random around some mean. Rather, they are seemingly composed of shot events with a variety of peak intensities and durations (Negoro et al. 1995). Number of analyses of X-ray light curves and optical AGN light curves show that power spectral density (PSD) is flat at lower frequencies (f) and is power-law ($\propto f^{-\alpha}$ with $\alpha \simeq 1-2$) at higher frequencies. The break frequency corresponds to a few second for BHCs and to a few years for AGNs. More, $1/f^{\alpha}$ fluctuations are ubiquitous in natural behavior, although their origins have been unsolved (Tajima & Shibata 1997; Cable & Tajima 1997). The significance of the $1/f^{\alpha}$ noise is that it contains a long-term memory (Press 1978). It has been a puzzle how $1/f^{\alpha}$ fluctuations can arise in black-hole accretion flows (or disks) under realistic circumstances.

Among number of suggestions for a possible mechanism of variability, the most promising one is magnetic flares (Wheeler 1977; Galeev, Rosner & Vaiana 1979). It has been established through the X-ray observations by the Yohkoh satellite that the solar flares are triggered by magnetic reconnection (Shibata 1996). In fact, solar soft X-ray variation exhibits $1/f^{\alpha}$ fluctuations (UeNo et al. 1997). Similarly, sporadic magnetic reconnection events which can occur in accretion flow may be responsible for the variability of black-hole objects, as well (Mineshige, Kusunose & Matsumoto 1995).

To prove this conjecture, we examine the three-dimensional data of global, MHD disk calculations first made by Machida, Hayashi & Matsumoto (1999). They calculated how magnetic field evolves in a rotating disk initially threaded by toroidal (B_{φ}) fields. Since no cooling is taken into account in computations, the simulated disk is advection-dominated (Kato, Fukue & Mineshige 1998), rather than radiation-dominated as in the standard disk. Then, the system we analyze corresponds to BHCs in the hard (low) state, in which fluctuations are largely enhanced and whose spectra can well be reproduced by advection-dominated flow (Narayan, McClintock & Yi 1996). Magnetic fields are amplified with time via a number of MHD instabilities together with differential rotation. The maximum field strength is determined either by field dissipation by reconnection or field escape from accretion flow via Parker instability. As a result, the mean plasma β, the ratio of gas pressure to magnetic pressure, finally reaches $\sim$10 irrespective of initial values

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of β. Locally, however, even low-β (< 1) regions appear; inhomogeneous structure arises consequentially [see also similar discussion by Abramowicz et al. (1992) but for non-magnetic cases].

In the present paper, we thus analyze temporal and spatial behavior of the accretion flow in order to clarify the origin of $1/f^{\alpha}$ fluctuations in black-hole objects. Results of the analysis are presented in the following section. The final section is devoted to discussions.

2. Temporal and Spatial Analysis of Accretion Flow

Figure 1 displays the light curves (A) and their PSDs (B) of the simulated disk obtained in the quasi-stationary state. Here, we assume that magnetic reconnection events contribute much to the energy output and thus calculated time variation of $\int \eta j^2 dV$ (with η and j being the electric resistivity and electric current density) integrated over almost whole disk. High frequency sides of the PSDs show nearly power-law decline with an index of $\alpha \sim 2$, in agreement with the observations. This is, in a sense, amazing that without any fine-tuning of parameters or special assumptions an MHD disk naturally gives rise to $1/f^{\alpha}$ fluctuations over three orders of frequency ranges. On the other hand, low frequency sides flatten at frequencies smaller than a reciprocal of several rotation timescale at a reference radius, which is also consistent with the observations (van der Klis 1995; Ulrich et al. 1997).

The appearance of $1/f^{\alpha}$ fluctuation, or more specifically, the presence of a long-term time correlation implies a long-distance spatial correlation in the distribution of magnetic fields. It is thus tempting to examine the spatial magnetic-field distribution. We specially pick up the quantity, j/ρ, the ratio of absolute value of electric current density to matter density, since it is a good indicator regarding a trigger of reconnection (Parker 1994; Ugai 1999). In fact, it is shown by MHD simulations that a fast reconnection, as is observed in solar flares, occurs when the electric resistivity becomes anomalously high in localized regions (Tajima & Shibata 1997). Such a local, anomalous resistivity can be achieved where electron drift velocity (which is proportional to j/ρ) exceeds a critical value (Yokoyama & Shibata 1995). Hence, any regions with high j/ρ values are all good candidates for a next reconnection site.

We plot in figure 2 a snapshot of the spatial j/ρ distribution on a horizontal plane slightly above the equatorial plane. The panel roughly demonstrates to what extent reconnected area expands, once reconnection is initiated somewhere. We notice that the distribution is quite inhomogeneous; patchy patterns are visible everywhere in figure 2. Importantly, there seems to be no typical size

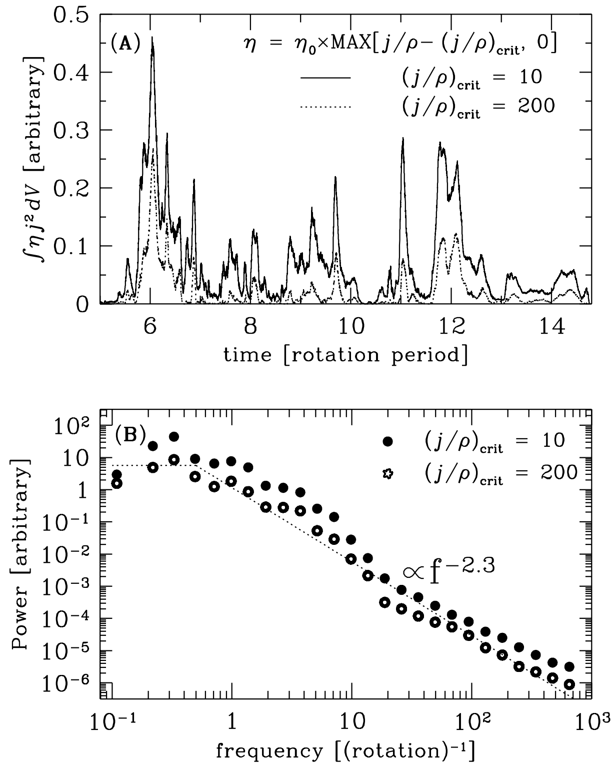

Fig. 1.. (**A**) Typical light curves of the simulated MHD disk. Here, we assume that radiation is predominantly due to field dissipation by magnetic reconnection, thus plotting the temporal variation of the ηj^2 integrated over almost whole disk. Electric resistivity η is assumed to be $\eta_0 \times \mathrm{MAX}[j/\rho - (j/\rho)_{\rm crit}, 0]$, since magnetic reconnection seems to happen where local j/ρ is larger than some critical value. (**B**) The power-spectral densities of light curves above. Dotted line is a least square fit with a broken power-law function for $(j/\rho)_{\rm crit}$ of 200 (open circles). It is of great importance to note that general behavior does not depend on the values of $(j/\rho)_{\rm crit}$. The unit of time is rotation period at a reference radius where the center of initial torus is located. The rotation timescale used here will correspond to a few seconds for BHCs and a few years for AGNs for in realistic situation.

of each patch. The presence of fractal structure is suspected.

To confirm this idea, we made a fractal analysis for the three-dimensional MHD disk with mesh point numbers of $(N_{\rm x}, N_{\rm y}, N_{\rm z}) = (100,100,25)$; namely we first mark the sites where j/ρ exceeds some critical value, $(j/\rho)_{\rm crit}$, and name any assembly of the marked sites clusters. We then count the numbers of clusters according to the cluster sizes (i.e., volume) for the data sets at five different timesteps and average the counts. The resultant distribution is plotted in figure 3 with thick lines. Surprisingly, the cluster size is distributed in a power-law fashion over three orders of magnitudes of cluster sizes, from a few larger clusters to numerous smaller clusters.

For comparison, we calculated a random distribution in the following way: we evaluate first how much fraction of the entire volume is covered with the marked sites in

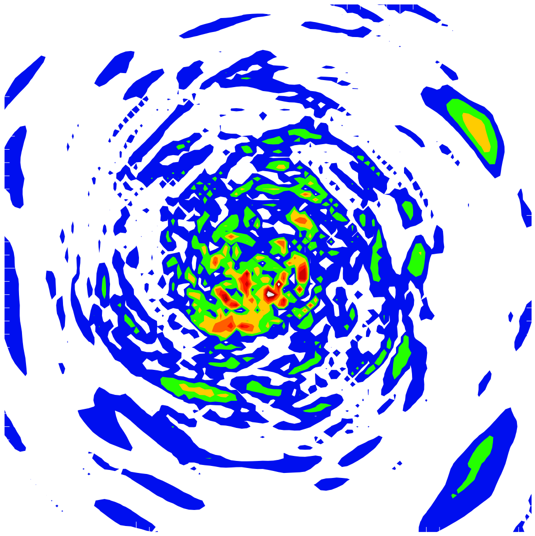

Fig. 2.. Color contour map of the (j/ρ) distribution on a horizontal plane slightly above the equatorial plane. Here, values of (j/ρ) where colors change are as follows; 70 from white to blue, 120 to green, 200 to yellow, and 300 to red, respectively.

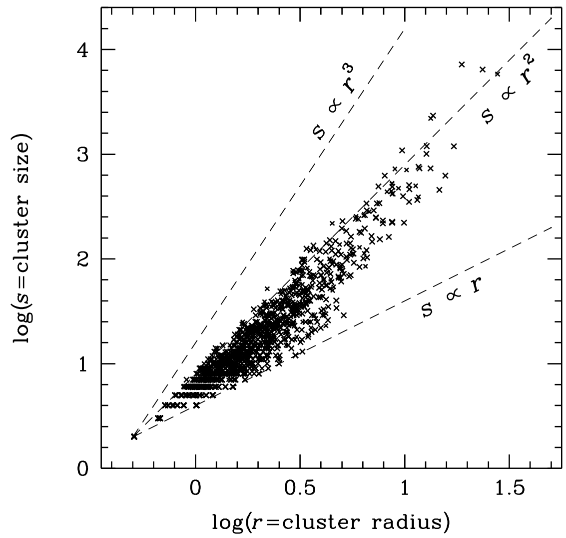

Fig. 4.. Relation between the volume (s) and mean radius (r) of each cluster. The lines of $s \propto r$, $s \propto r^2$ and $s \propto r^3$ are also depicted. The least square fit shows $s \propto r^D$ with $D \sim 1.9$.

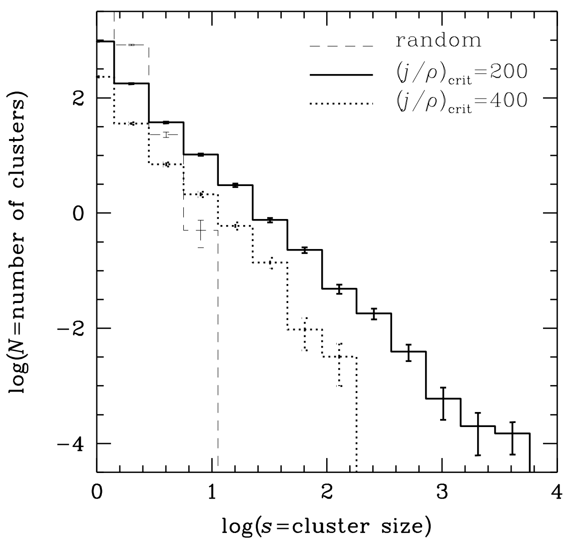

Fig. 3.. Histograms of clusters with $j/\rho > (j/\rho)_{\rm crit} = 200$ (by the thick solid lines) and those with $j/\rho > 400$ (by the thick dotted lines) as functions of the cluster size (volume). Power-law distribution is realized over three orders of cluster sizes; roughly, $D(s) \propto s^{-2}$. For comparison, we also plot the same but for a random distribution (by the thin dashed lines). Obviously, large clusters are missing.

figure 2, finding about 5.5% for $j/\rho > 200$. We next put a random number between 0 and 1 in each site of the three-dimensional box, and mark the sites where the number exceeds 0.945. We then repeat the same procedure done for the MHD disk and plot the resultant histogram in figure 3 with the thin dashed line. Clearly, there are no very big clusters in the random model. In other words, a long-distance j/ρ correlation is lost there.

Magnetic fields in the disk have fractal structure. To find a fractal dimension, we plot a size (volume) of each cluster as a function of its mean radius in figure 4. Here, the cluster mean radius is defined as $r \equiv \frac{1}{s}\sum_i^s |\mathbf{r}_i - \mathbf{r}_{\rm CM}|$ with $\mathbf{r}_{\rm CM} = \frac{1}{s}\sum_i^s \mathbf{r}_i$ for each cluster with a size of $s(>1)$, where $\mathbf{r}_i$ is the coordinates of the i-th site belonging to the cluster. From the fitting, we find roughly $s \propto r^D$ with $D = 1.9$.

3. Discussions

Then, we address two key questions: what relates the fractal structure to the temporal fluctuation in the simulated MHD disk? How can such a fractal distribution arise? Turbulence, in general, is known to exhibit fractal behavior (Procaccia 1984) and this fact may be related to our present finding, but predicted fractal dimension is $D \sim 2.6$, differing from that of the present case. Alternatively, we note the notion of self-organized criticality (SOC; Bak 1996; Jensen1998), one of the most attractive concepts developed in the study of complex systems.

Bak, Tang & Wiesenfeld (1988) proposed a sand-pile model to describe a system exhibiting $1/f^\alpha$ fluctuation. Suppose that we fall sand particles one after another on a table. Fallen sand particles will form a pile, onto which another sand particle will be added. When a slope of the pile in either direction exceeds a critical value, an avalanche occurs and sand particles will slide down in that direction. Then, the system spontaneously evolves to and stays at SOC. In our case, addition of a sand

particle corresponds to energy input to magnetic fields, while the critical slope corresponds to the critical j/ρ over which energy dissipation occurs via reconnection (Mineshige, Takeuchi & Nishimori 1994).

For systems in a SOC state, long-distance spatial communication among different sites is naturally built up and it yields long-term time correlation. If each flare light curve is expressed by a time-symmetric profile with exponential grow and decay about $t = 0$, $L(t) \propto \exp(-|t|/\tau)$ with τ ($\leq \tau_{\max}$) being constant, its PSD is $P_\tau(f) \propto \epsilon^2/(1+4\pi^2 f^2\tau^2)^2$. If the energy ($\epsilon$) of each flare is distributed as $N(\epsilon) \propto \epsilon^{-p}$ and each flare duration is related to energy as, $\epsilon \propto \tau^D$, the total PSD becomes

$$\begin{aligned} P(f) &= \sum_{\tau}^{\tau_{\max}} P_\tau(f) N(\tau)\Delta\tau \\ &\propto \sum_{\tau}^{\tau_{\max}} \frac{\tau^{2D}}{(1+4\pi^2 f^2\tau^2)^2}(\tau^D)^{-p}\frac{d\epsilon}{d\tau}\Delta\tau, \\ &\longrightarrow P(f) \propto \left(\frac{1}{f}\right)^{(3-p)D} F(f) \qquad (\Delta\tau \to 0). \quad (1) \end{aligned}$$

Here, $F(f)$ $[= \int_0^{2\pi\tau_{\max} f} dx (x^{(3-p)D-1})/(1+x^2)^2]$ is a slowly varying function of f for $f > 1/(2\pi\tau_{\max})$. We may regard $\epsilon \propto s$ (volume of a clump) and $\tau \propto r$ (mean radius of a clump). Then, we find from the simulation that $p \sim 2$ and $D \simeq 2$ (see figure 4). Hence equation (1) leads $P(f) \propto f^{-2}$, in agreement with the numerical result (see figure 1). That is the reason why the fractal magnetic field produces $1/f^\alpha$ fluctuations (Takeuchi, Mineshige & Negoro 1995; Kawaguchi et al. 1998).

One of the most conspicuous natures of the SOC is its ubiquity; namely, it is supposed to describe various non-equilibrium open systems, such as earthquakes, forest fires, evolution of biological species, and traffic flows (Bak 1996). In astrophysical context, it is important to note that coronal magnetic fields in the Sun are suggested to be in a SOC state (Lu & Hamilton 1991; Vassiliadis et al. 1998) as well, thus exhibiting power-law occurrence rate of flares and $1/f^\alpha$ fluctuations in solar flare curves (UeNo et al. 1997).

Gamma-ray bursts (GRBs) also occasionally exhibit $1/f^\alpha$ fluctuations (Beloborodov, Stern & Svensson 1998). In some models of GRBs which involve merger of two compact objects (white dwarf, neutron star, and black hole) a sort of an accretion disk is thought to be formed by debris of one component around the other (Mészáros 1999). The situation could be similar. We thus expect frequent reconnection events with a smooth size (amplitude and duration) distribution to occur in GRBs, which will give rise to $1/f^\alpha$ fluctuations (Panaitescu, Spada & Mészáros 1999). Likewise, any other magnetic systems, regardless of system size, may show similar effects.

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References

Abramowicz, M. A., Lanza, A., Spiegel, E. A., & Szuskiewicz, E. 1992, Nature, 356, 41
Bak, P., Tang, C., & Wiesenfeld, K. 1988, Phys. Rev. A, 38, 364
Bak, P. 1996, How Nature Works (Springer-Verlag, New York)
Beloborodov, A.M., Stern, B.E., & Svensson R. 1998, ApJ, 508, L25
Cable S., & Tajima T. 1997, Comm. Plasma Phys. Controlled Fusion, 18, 145
Galeev, A.A., Rosner, R., & Vaiana, G.S. 1979, ApJ, 229, 318
Jensen, H.J. 1998, Self-Organized Criticality (Cambridge Univ. Press, Cambridge)
Kato, S., Fukue, J., & Mineshige, S. 1998, Black-Hole Accretion Disks (Kyoto Univ. Press, Kyoto)
Kawaguchi, T., Mineshige, S., Umemura, M., & Turner, E.L. 1998, ApJ, 504, 671
Lu, E.T., & Hamilton, R.J. 1991, ApJ, 380, L89
Machida, M., Hayashi, M. & Matsumoto R. 1999, submitted to ApJL
Mészáros, P. 1999, To appear in Black Holes and Gravitational Waves. (ed Nakamura, T., Supplement of Prog. Theor. Phys)
Mineshige, S., Takeuchi, M., & Nishimori, H. 1994, ApJ, 435, L125
Mineshige, S., Kusunose, M., & Matsumoto, R. 1995, ApJ, 445, L43
Narayan, R., McClintock, J. E., & Yi, I. 1996, ApJ, 457, 821
Negoro, H., Kitamoto, S., Takeuchi, M., & Mineshige, S. 1995, ApJ, 452, L49
Panaitescu, A., Spada, M., & Mészáros, P. 1999, submitted to ApJ, astro-ph/9905026
Parker, E. N. 1994, Spontaneous Current Sheets in Magnetic Fields (Oxford Univ. Press, New York) p50
Press, W.H. 1978, Comm. Astrophys., 7, 103
Procaccia, I. 1984, Journal of Statistical Physics, 36, 649
Shibata, K. 1996, Adv. Space Res., 17, (4/5)9
Tajima, T., & Shibata, K. 1997, Plasma Astrophysics (Addison Wesley, Massachusetts,) ch2.6
Tajima, T., & Shibata, K. 1997, Plasma Astrophysics (Addison Wesley, Massachusetts) p237
Takeuchi, M., Mineshige, S., & Negoro, H. 1995, PASJ, 47, 617
UeNo, S. et al. 1997, ApJ, 484, 920
Ugai, M. 1999, Phys. Plasma, 6, 1522

Ulrich, M.-H., Maraschi, L., Urry, C.M. 1997, ARA&A, 35, 445

van der Klis, M. 1995, in X-Ray Binaries, ed W.H.G. Lewin et al. (Cambridge University Press, Cambridge) p252

Vassiliadis, D., Anastasiadis, A., Georgoulis, M., & Vlahos, L. 1998, ApJ, 509, L53

Wheeler, J.C. 1977, ApJ, 214, 560

Yokoyama, T. & Shibata, K. 1995, Nature, 375, 42